MAT 152 - Pre-Calculus (Survey of Functions II)

General Information

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Department Mathematics
Course Prefix MAT
Course Number 152
Course Title Pre-Calculus (Survey of Functions II)

Course Information

Catalog Description This course is a continuation of the study of families of functions from those included in MAT 145, Survey of Functions I. Exponential, logarithmic, trigonometric/sinusoidal, and rational functions are analyzed in depth. Embedded within the study of each of these families are composition, decomposition, and the creation of inverse functions. An introduction to limit notation is used to analyze long range behavior, horizontal and vertical asymptotes, as well as removable discontinuities. The use of realistic applications and modeling with these families of functions is an essential element of this course. Emphasis on multiple methods of solving equations (algebraic, graphic, and numeric) is included as are multiple representations (algebraic, graphic, numeric, and verbal) of mathematical information.

Credit Hours 3
Lecture Contact Hours 4
Lab Contact Hours 0
Other Contact Hours 0
Grading Scheme Letter

Prerequisites

MAT 145 or Placement into Math Level 3
Co-requisites
None

First Year Experience/Capstone Designation
This course DOES NOT satisfy the outcomes applicable for status as a FYE or Capstone.

SUNY General Education
This course is designated as satisfying a requirement in the following SUNY Gen Ed category
Mathematics (and Quantitative Reasoning)

FLCC Values

Institutional Learning Outcomes Addressed by the Course
Inquiry and Interconnectedness

Course Learning Outcomes

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1. Model realistic scenarios using exponential, logarithmic, sinusoidal, and rational functions.
2. Use algebraic skills to combine, compose, decompose and invert functions.
3. Use limit notation to explain long range behavior, asymptotes, and removable discontinuities of functions.
4. Solve equations algebraically, graphically, and numerically (via tables) and evaluate the result for reasonableness.

Outline of Topics Covered

General Outline of Topics Covered:

1. Common to all function families below (embed throughout the course)
   
   I. a. Understanding and using function notation including variables other than x and y.
   
   b. Function evaluation
c. Characteristics of their graphs (increasing, decreasing, concavity, etc.)

d. Choosing bounds to graph functions in an appropriate window using a graphing calculator

e. Finding the zeros and vertical intercept algebraically and graphically

f. Solving for the input of a function given an output algebraically

g. Solving inequalities related to functions graphically

h. Interpreting the realistic meaning of the inputs and outputs, zeros and y-intercept

i. Stating domain and range: both abstract and realistic

j. Effects (graphical, algebraic, and verbal) of transformations

k. Defining a formula for a function from a given graph, table, and verbal expression

l. Using limit notation to describe the end behavior of functions

m. Calculating and interpreting average rate of change (AROC).

2. Functions (embed throughout the course)

   a. Defining new functions by performing binary operations on the rules of existing functions

   b. Evaluating functions with expressions (possibly via difference quotient)

   c. Composition of functions

      i. Evaluation

      ii. Finding formulas algebraically (for all families of functions in the course)

      iii. Domain and range

      iv. Decomposition

   d. Inverse Functions

      i. Definition of a function (review)

      ii. Determining when a function is invertible over its entire domain
1. Restricting the domain to make functions invertible

   iii. Notation \([x = f^{-1}(y)]\) and interpretation

   iv. Constructing the formula of an inverse of a function

   v. Determining if two functions are inverses:
      1. Graphically through reflections in \(y = x\)
      2. Algebraically as individual functions and through composition

3. Exponential Functions (concentration on base e)
   a. General form, properties of, and modeling with exponential functions (review)
   b. Convert growth factors between time scales (e.g.: monthly to annual percent rate of change and vice versa)
   c. Development of base e from compound interest formula
   d. Continuous growth form
   e. Converting between different bases
   f. Limits at positive/negative infinity
   g. Applications with base e: (e.g.: time value of money, population growth, doubling time, half-life, etc.)

4. Logarithmic Functions
   a. Common, natural, and other base logarithms
   b. Converting between exponential and log forms
   c. Use single-sided limit notation to describe the vertical asymptote
   d. Change of base formula
   e. Properties of logarithms
      i. Relationship to exponent rules
ii. Use to simplify and expand algebraic expressions

iii. Use to solve exponential equations

f. Solving logarithmic equations

g. Applications: (e.g. : comparing orders of magnitude, graphing using log scales, decibels, Richter and pH scales, etc.)

5. Trigonometric Functions

a. Sine/cosine/tangent of an angle (review)

b. Center-radius form of a circle

c. Determining (x,y) coordinates on a circle with a given radius and angle measure.

d. Defining radian measure through arc length

e. Converting between radian and degree measure

f. Inverse trigonometric functions

g. Solve equations with sine, cosine, and tangent using radian measure

h. Pythagorean identity \( \sin^2(\Theta) + \cos^2(\Theta) = 1 \)

i. Sinusoidal functions

   i. Finding period, amplitude, frequency, and shift (vertical and horizontal) algebraically and graphically

   ii. Interpreting period, amplitude, frequency, and shift (vertical and horizontal) algebraically and graphically

   iii. Creating sinusoidal models from graphs, tables, or verbal descriptions

j. Applications: (e.g. : Ferris wheels, daylight hours, pendulums, etc.)

6. Rational Functions

a. Polynomial functions (review)

b. Algebraic manipulation into
\[ r(x) = \frac{p(x)}{q(x)} \] form (including manipulation of complex fractions)

c. Long term behavior from ratio of leading terms

d. Use limit notation to describe asymptotic behavior (vertical, horizontal, slant - optional), end behavior and removable discontinuities (holes)

e. Short term behavior: identifying (algebraically and graphically) y-intercept, zeros, undefined values and connection to vertical asymptote(s)

f. Applications (e.g.: average cost, concentration, etc.)